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CIRCUIT DUALS ON PLANAR TRANSMISSION MEDIA*

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Abstract

Lumped element and multiconductor transmission line configurations in a plane have dual circuit representations, as shown by coplanar and twinstrip transmission lines. Duality relations and circuits having possible applications to MIC and MMIC are given.

Introduction

This paper describes some analytical and experimental work conducted in support of original demonstrations by Stegens,[1] that the coplanar waveguide (CPW) is a practical transmission medium having certain advantages over microstrip, where cost and size of microwave circuits are important. CPW can also be a preferred medium for MMIC for millimeter wavelengths at which microstrip requires substrates that may be impractically thin. Finally, twin-strip (TS) transmission line, the physical dual of CPW, has significant potential for balanced MIC and MMIC solid-state power amplifiers[2] and for circuitry driving phased-array dipoles.

Electromagnetic duality,[3] often referred to as Babinet's Principle,[4] has been used previously to relate[5] the transmission line parameters of coplanar and TS lines shown in Figure 1.

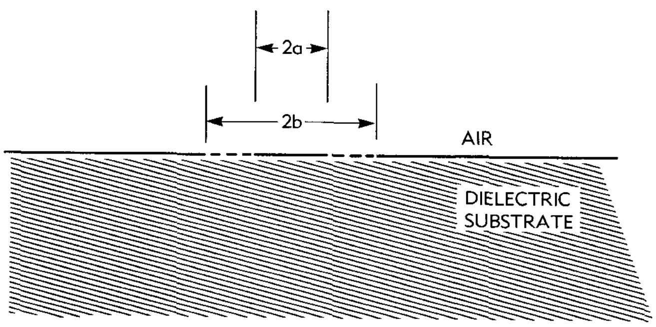


Figure 1. Coplanar (CPW) and Twin Strip (TS) Transmission Lines

This paper extends the applications of electromagnetic duality to show that any planar circuit and its physical dual are circuit theory duals as well, provided that they can be described reasonably as quasi-TEM structures, that is, in terms of voltage and current. This insight is applied to discontinuity analysis and to various multiconductor structures such as filters, couplers, and transitions.

Theory

Consider a charge and current-free region bounded by electric and magnetic walls. If E_1 and H_1 represent a solution to the boundary value problem, then $E_2 = -\eta H_1$ and $H_2 = E_1/\eta$ represent a solution for the same boundaries, but with electric and magnetic walls interchanged.[3] If the boundaries of E_1 and H_1 are denoted as a circuit, then the boundaries of E_2 and H_2 are considered the physical dual[5] of the circuit.

At microwave frequencies, practical approximations to dual circuits are based on a thin metal plane that has various openings. This achieves magnetic walls by symmetry. CPW and TS are approximations to this concept because their inhomogeneous dielectrics make them dispersive. However, they can be characterized by an effective homogeneous dielectric constant, and they are considerably less dispersive than microstrip of comparable dimensions and substrate dielectric constant[6]; thus the quasi-TEM assumption may be used. CPW and TS have an effective dielectric constant ε_e of

$$\varepsilon_e = \frac{\varepsilon_r + 1}{2} \tag{1}$$

where ε_r is the relative dielectric constant of the substrate.

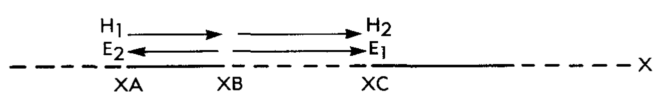
As shown by Stegens,[1] it is generally accepted in CPW circuit design that transverse symmetry and ground-potential continuity are essential to avoid coupling to slot mode and other undesired modes. Such precautions are assumed in this paper.

Figure 2 represents a cross section at an arbitrary terminal plane on a planar circuit or its physical dual, not necessarily realized as TS or CPW. It is assumed here that voltage V_1 between two conductors and current I_1 on a conductor for the circuit can be found uniquely, and similarly for voltage V_2 and current I_2 for the dual circuit, relative to point x_b. The result is that the impedance of one circuit is related to the impedance of its physical dual by

$$Z_1 Z_2 = \frac{\eta^2}{4} \tag{2}$$

as indicated in Figure 2.

Duality Relations: $E_2 = -\eta H_1 \qquad H_2 = E_1/\eta$



$$V_1(XB-XC) = \int_{XB}^{XC} E_1 \cdot dX \qquad V_2(XB-XA) = -\int_{XA}^{XB} E_2 \cdot dX$$

$$I_1(XB-XA) = 2\int_{XA}^{XB} H_1 \cdot dX \qquad I_2(XB-XC) = 2\int_{XB}^{XC} H_2 \cdot dX$$

Substitute duality relations, yielding

$$V_2 = \eta I_1/2 \qquad I_2 = 2V_1/\eta$$

Thus; $Z_1 = \eta^2/4 \; Y_2 \quad Z_2 = \eta^2/4 \; Y_1$

Figure 2. Electromagnetic Duality Implies Circuit Theory Duality

*This paper is based upon work performed at COMSAT Laboratories under the sponsorship of the Communications Satellite Corporation.

Because this relation characterizes circuit theory duals[7,8] and must hold for any planar circuit configuration at any terminal plane subject to the reasonableness of the TEM assumption, it follows that the circuit and its physical dual are also circuit theory duals regardless of the complexity of the circuit.

In MKS units,

$$\frac{\eta^2}{4} = \frac{35475.7}{\varepsilon_e} \tag{3}$$

Application to Lumped Elements

Equation (2) shows that an inductance, L, in one circuit becomes a capacitance, C, in its dual. The relation is

$$L_1(\text{nH}) = \frac{35.48}{\varepsilon_e} C_2\ (\text{pF}) \tag{4}$$

for any inductance and capacitance in dual planar circuits.

Assume that it is desired to know the capacitance of a series gap in the center conductor of CPW, as shown in the upper half of Figure 3. This is a difficult three-dimensional problem that perhaps would be solved by numerical techniques. But suppose that the dual problem of the inductance of a connecting strip between TS lines (given by the same upper sketch of Figure 3) is solved instead. It can be shown that the inductance of an uncoupled thin strip of width w and length ℓ is given to a good approximation by[9]

$$L = \frac{\mu\ell}{2\pi}\left[p - \sqrt{1+p^2} + \ell n\,\frac{1+\sqrt{1+p^2}}{p}\right] \tag{5}$$

where $p = w/4\ell$, and $w, \ell \ll \lambda$.

Substitution of equation (5) into equation (4) immediately gives the capacitance of the gap in CPW. The same technique can be applied to the dual circuits of the lower sketch of Figure 3.

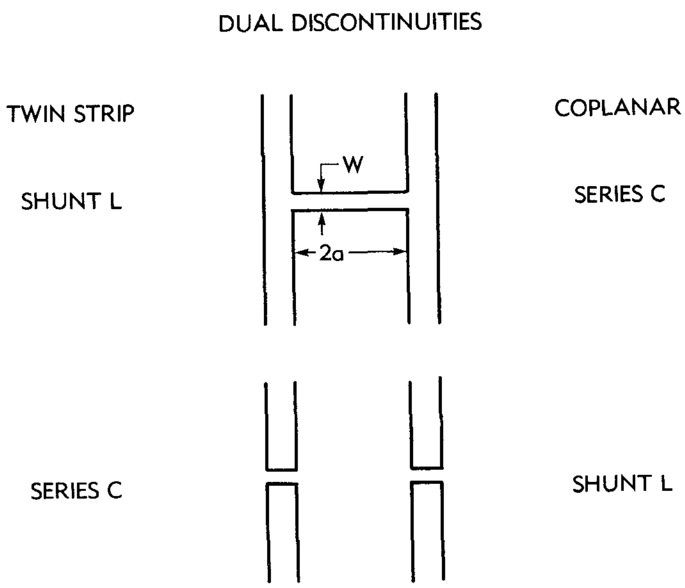


Figure 3. Dual Discontinuities

Measurements demonstrating the applicability of equations (4) and (5) will be presented at the Symposium.

Suppose now that part of a CPW gap, such as indicated in Figure 4, is given a thin deposit of material having surface resistivity ρ_1, and its TS dual is given a deposit of surface resistivity ρ_2. Then, resistance R and conductance G for the two two circuits are

$$R_1 = \rho_1 w/2a \text{ and } G_2 = w/2a\rho_2 \tag{6}$$

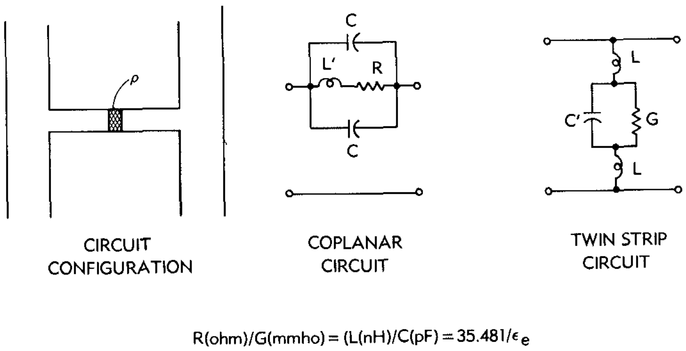


Figure 4. Lumped Element Planar Circuit Duality

These relations satisfy equation (2) if

$$\rho_1\rho_2 = \eta^2/4 \quad ; \tag{7}$$

the shape factor, w/2a drops out.

If equation (7) is met,

$$R_1 = \frac{\eta^2}{4} G_2 \tag{8}$$

and R, L, and C all satisfy the dual relationships of the two circuits (assuming lossless conductors and dielectric).

The above ideas are summarized in Figure 4. The inductances are found from equation (5), capacitances are found from dual inductances transformed by equation (4), and resistances assume that equation (7) has been satisfied. It can be seen that these circuits are true duals.

Application to Transmission Lines

Figure 5 illustrates a means of transforming between CPW and TS.[2] Observe that this sketch shows two distinct transformers, depending on which areas are metal and which dielectric. The transforming section in the middle can be considered as CPW with narrow ground planes (unbalanced mode) or as TS with a center ground strip (balanced mode) for one assumed metallization. For the other metallization, the center section can be considered a CPW with a split center conductor (unbalanced mode) or TS with external grounds (balanced mode). Although none of these multiconductor transmission lines is conventional CPW or TS, the relation between the CPW-like (unbalanced) line characteristic impedance, and the TS-like (balanced) line characteristic impedance is given by equation (2).

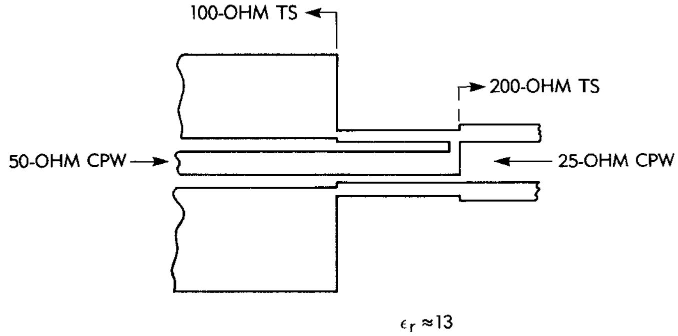


Figure 5. CPW to TS Transition

Figure 6 shows a family of transversely symmetrical, coupled-line filter sections in either CPW or TS, depending on which areas are metallized. The balanced-strip TEM equivalent circuits shown in Figure 6 are applicable for two reasons: CPW and TS have small dispersion; and the coupled sections are designed using a special mapping[10] that imposes equal capacitance to ground for the inner pair and outer pair of conductors. Thus, these sections are the equivalent of coupled sections of equal strip width in stripline, and so CPW and TS filters can be designed readily from the stripline formulas of Matthaei, Young, and Jones.[11] The impedances (of either even or odd modes) of these coupled sections are related as between CPW and TS by equation (2).

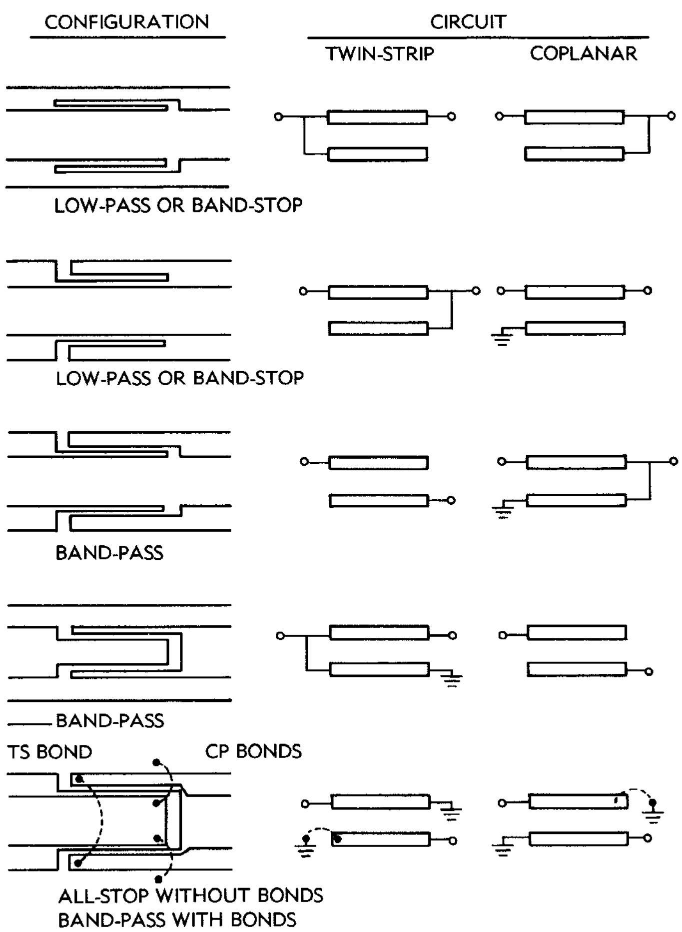


Figure 6. Dual Parallel Coupled Filter Sections

It is interesting that because the coupled CPW and TS configurations are duals, each pair of coupled TEM circuits shown in Figure 6 must also be duals; thus, previously unrecognized coupled-line duals appear. Also, because the dual of the dual of a circuit must be equivalent to the circuit, Figure 6 can be used to show previously unrecognized electrical equivalences between parallel coupled circuit configurations.

A 2-resonator parallel-coupled filter was designed and built on coplanar line in accordance with References 10 and 11 using one detailed section of Figure 6. The performance was in reasonable agreement with that predicted on a lossless TEM-line basis.

References

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